Studying the Coefficient of Restitution of a Tennis Ball and Tennis Racket at Differing Impact Points

1. Introduction

I have been playing tennis since the fifth grade and have noticed that my coaches have always instructed me to hit the ball at my racket's center. They would argue that it was the easiest place to control the ball and, hence, the most optimum location for impact. Intrigued by this fact, I decided to put my knowledge as an IB Physics student to the test and research the properties of tennis rackets, eventually learning that they have a point called the "sweet spot" existing at their center.

In the sport of tennis, a racket's sweet spot is where if impacted, the force transmitted to the hand is sufficiently small. It is normally the best place to hit a ball at, as it causes less strain for the player and is easier to control.[1] However, as I have continued to play throughout the years, I have noticed that the sweet spot is not always the best place to hit a ball when needing to hit shots that require more power. For example, when returning a serve or hitting a volley, I have found more success hitting the ball at a location moderately below the center of my racket. Wanting to further explore this phenomenon and discover where my racket would generate the most powerful shots when hit, I was led to my following **research question:**

How does varying a tennis ball's impact point ($r,\ \theta$) on a tennis racket affect their COR, when the tennis ball's initial drop height (m), the tennis ball's temperature, and the tennis racket's temperature are kept constant for the same tennis ball and tennis racket?

In my Physics IA, an impact point refers to the point on the tennis racket head at which is hit by the tennis ball It is represented by both the impact point's distance from the tennis racket head's center (which will be referred to as the radius of the impact point and will be represented in the unit meters/ m) and the angle of the impact point to the horizontal axis of the tennis racket head (which will be represented in the unit degrees/ °). Also, COR refers to the coefficient of restitution.

2. Background Information

2.1 Defining COR

The COR is the ratio of the final velocity to the initial velocity between two objects after their collision. It was developed by Issac Newton and is used in Newton's Law of Restitution, which states that the post-collision speed of two objects depends on the materials from which they are made. The COR is used to represent what different types of colliding objects are made off. [2]

[1] *Tennis Raquet Physics*. http://www.physics.usyd.edu.au/~cross/tennis.html. Accessed 5 Aug. 2023.
[2] Admin. "Coefficient of Restitution - Definition, Range of Values, Examples, Solved Problems." *BYJU'S*, 13 Apr. 2021, https://byjus.com/jee/coefficient-of-restitution/.

$e = \frac{relative\ speed\ after\ collision}{relative\ speed\ before\ collision}$

In my experiment, I will define the COR at a specific impact point as $COR(r, \theta)$. Assuming that the tennis ball's velocity when dropped will be 0, we can use the Law of Conservation of Energy to get:

- $v_{initial} = \sqrt{2gh_{initial}}$
- $v_{final} = \sqrt{2gh_{final}}$

Therefore, $COR(r, \theta)$ can be measured as $\sqrt{\frac{h_{final}}{h_{initial}}}$ as this is also equal to $\frac{v_{final}}{v_{initial}}$

(We will use this equation to measure $COR(r, \theta)$ in our experiments).

2.2 Energy Lost During Impact of Tennis Balls and Tennis Rackets

The energy lost (EL) at the impact point on the racket depends on the number of vibrations of strings at the impact point. The more string vibrations there are, the more energy is lost from the "collision system" in the forms of heat and sound. String Vibrations depend on:

- The net tension of strings ($T(r, \theta)$) at the impact point. High net tension means less vibration and therefore less energy loss.
- The impact point's distance from the tennis racket's vibration node ($D(r, \theta)$). A vibration node is a point on the tennis racket that never moves when a wave passes through it. It is where the amplitude of vibrations becomes zero. When the ball hits the vibration node, the tennis racket hardly bends, leading to less/ no string vibrations. Therefore the closer the vibration node is to the impact point, the less energy loss in the "collision system". [3]

2.3. Relationship between COR and Energy Loss

As $COR(r, \theta)$ is the quotient of the tennis ball's final and initial velocity, it is also equal to the $\sqrt{\frac{KE_f}{KE_i}}$ or the square root of the quotient of the final kinetic energy and initial kinetic energy (as kinetic energy = $\frac{1}{2}mv^2$). Thus, if energy loss occurs in the system, $COR(r, \theta)$ decreases.

Using the information above, we can generalize that $COR(r, \theta) \propto (\frac{1}{\sqrt{EL}})$ in the collision system.

Therefore, $COR(r, \theta) \propto (\sqrt{T(r, \theta)})$ and $COR(r, \theta) \propto (\frac{1}{\sqrt{D(r,\theta)}})$, which finally means that

$COR(r, \theta) \propto \sqrt{\frac{T(r,\theta)}{D(r,\theta)}}$.

[3] Bocchi, Fabio. *The Physics of Tennis Racket Sweet Spots*. 27 Aug. 2015, https://www.comsol.com/blogs/the-physics-of-tennis-racket-sweet-spots/.

3. Hypothesis

While keeping the tennis ball's initial drop height, the tennis ball's temperature, and the tennis racket's temperature constant for the same tennis ball and tennis racket, I hypothesize that their COR will increase as the square root of the quotient of the net tension of the strings at the impact point ($T(r, \Theta)$) and the impact point's distance from the vibration node ($D(r, \Theta)$) increases.

Figure 1: Area of Maximum COR

Using my previous knowledge of tennis, I also hypothesize that the point of maximum COR will be directly under the center of the tennis racket (or near its throat), since this is where the racket is both the stiffest and very close to the vibration node.

4. Variables

Variable Type	Variable Name	Unit	How to Measure / How to Control
Independent Variable	1. The radius of the impact point (r)	Meters (m)	I will use a ruler to measure the radius of the impact point.
	2. The angle of the impact point with respect to the horizontal axis of the racket (Θ)	Degrees (°)	I will use a protractor and align it to the horizontal axis of the racket to measure the angle of the impact point.
Dependent Variable	1. Coefficient of Restitution (COR)	No unit	I will first find the quotient of the tennis ball's final bounce height and its constant initial drop height. I will then calculate the square root of this quotient to find COR.

Control Variable	1. The Initial Drop-Height of the Ball ($h_{initial}$)	Meters (m)	I will use a tape-measure to measure the ball's initial drop height and ensure that the control variable is constant across all trials. **Why should it be kept constant?** If I were to use differing drop-heights for the tennis ball, it's energy at impact would be different, possibly causing its bounce height to differ. This would make it difficult to accurately measure the COR as the ball's bounce height would be dependent on other factors as well, such as the speed of the ball at impact. **Value:** $2\ ft\ =\ 0.610\ m$ **Uncertainty:** 0.03125 $inches$ = 0.001 m
	2. The tennis ball and tennis racket's temperature	Degrees Celcius ($°C$)	I will conduct the experiment at once in the same room. I will close all windows, doors, and turn off the fan to ensure there is no significant change in temperature. **Why should it be kept constant?** Significant shifts in temperature can cause the tennis ball and tennis racket's physical properties to change. E.g, an increase in temperature can lead to the expansion of strings in the tennis racket, potentially changing the COR at different impact points and skewing the results of the experiment. **Value:** Room Temperature

5. Apparatus

Instrument	Purpose	Range and Least Count
1. Tape measure	Measure bounce height of the tennis ball (h_{final}).	(0 m to 8 m) $\pm$ 0.001 $meters$
2. Ruler	Measure the impact points' radius/ distance of the tennis racket's center (r).	(0 $inches$ to 12 $inches$) $\pm$ 0.031 inches
3. Protractor	Measure the angle between the impact point and the horizontal axis of the tennis racket (Θ).	(0° to 180°) $\pm$ 0.500°

6. Materials

Instruments	Purpose
1. Heavy Duty G-Cramps and Newspaper Foldings	To secure the tennis racket to the desk.
2. Tennis Racket (Wilson Blade) and Tennis Ball	The two pieces of sports equipment whose coefficient of restitution is being measured (these were kept the same throughout the entire experimental process).
3. iPhone	To record a video of the ball's motion.
4. Labeled Paper Strip	To identify the ball's impact points.

7. Experimental Setup

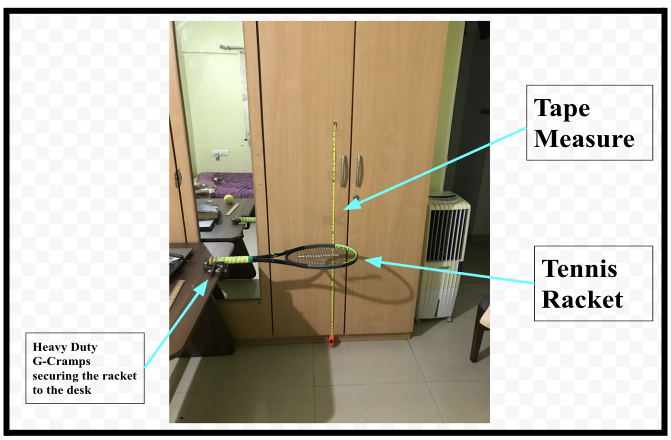

Figure 2: Main Experimental Setup

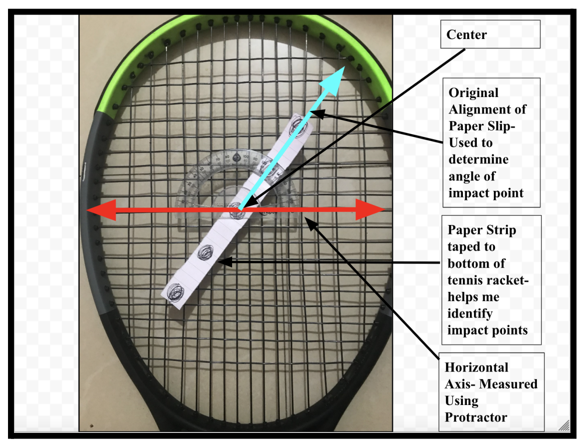

Figure 3: Identification of Impact Angles and Points

8. Method

I first used heavy duty g-clamps and newspaper foldings to secure my tennis racket to a table. After this, I attached a tape-measure to a cupboard door and aligned the table and tennis racket in such a way that I could record the tennis ball's maximum bounce height using my iPhone. To ensure that I was dropping the tennis ball at the correct impact point during each trial, I taped a precisely measured paper cutout, which I had labeled at equidistant points using a ruler, to the bottom of the tennis racket. Using a protractor, I always aligned this paper cutout at the specified angle of the impact points to the racket's horizontal axis. I performed and repeated this entire process for multiple trials so that I could easily represent the impact point's different radii. It is also important to mention that I used a ruler and protractor to find the center and horizontal axis of the tennis racket respectively. By measuring the width and height of the tennis racket head and dividing these two values by 2, I was able to obtain the coordinates for my tennis racket's center.

After setting up my experiment, I dropped a tennis ball from an initial drop height of two feet and recorded a video of its motion for multiple trials. Plugging this video into the *Try Tracker* software by *Physlet*, I measured its highest bounce height. This bounce height will then be plugged into the equation $COR = \sqrt{\frac{h_{final}}{h_{initial}}}$ to calculate the coefficient of restitution of the tennis ball-tennis racket system.

9. Raw Data

Table 1: Trial Measurements of the Bounce Height for respective Angles and Radii of the Impact Points of the tennis balls

Angle Between the Impact Point and the Horizontal Axis of the Tennis Racket (Θ in °) $\Theta \pm 0.500°$	**Radius of the Impact Point (r in $inches$) $r \pm 0.031$**	**Maximum Bounce Height (h_{final} in $meters$) $h_{final} \pm 0.001\ meters$**				
1. 72°	1. -3.200	0.127	0.124	0.132	0.132	0.135
	2. -1.600	0.149	0.157	0.150	0.150	0.152
	3. 0.000	0.127	0.127	0.124	0.127	0.127
	4. 1.600	0.076	0.081	0.071	0.070	0.079
	5. 3.200	0.064	0.070	0.070	0.064	0.064
2. 90°	1. -3.200	0.165	0.163	0.170	0.165	0.163
	2. -1.600	0.146	0.140	0.142	0.146	0.146
	3. 0.000	0.127	0.127	0.127	0.127	0.127
	4. 1.600	0.102	0.102	0.097	0.095	0.102
	5. 3.200	0.051	0.051	0.051	0.053	0.048
3. 125°	1. -3.200	0.191	0.188	0.191	0.187	0.191
	2. -1.600	0.140	0.144	0.145	0.142	0.137
	3. 0.000	0.102	0.127	0.121	0.127	0.127
	4. 1.600	0.076	0.081	0.080	0.076	0.080
	5. 3.200	0.038	0.041	0.038	0.038	0.038

165°	1. -3.200	0.064	0.066	0.070	0.070	0.064
	2. -1.600	0.114	0.117	0.114	0.121	0.112
	3. 0.000	0.127	0.127	0.122	0.122	0.133
	4. 1.600	0.089	0.091	0.091	0.089	0.089
	5. 3.200	0.025	0.028	0.028	0.025	0.025
180°	1. -3.200	0.025	0.032	0.025	0.025	0.028
	2. -1.600	0.102	0.099	0.099	0.102	0.102
	3. 0.000	0.127	0.121	0.132	0.131	0.127
	4. 1.600	0.095	0.097	0.095	0.093	0.097
	5. 3.200	0.051	0.048	0.047	0.048	0.048

9.1 Try Tracker By Physlet:

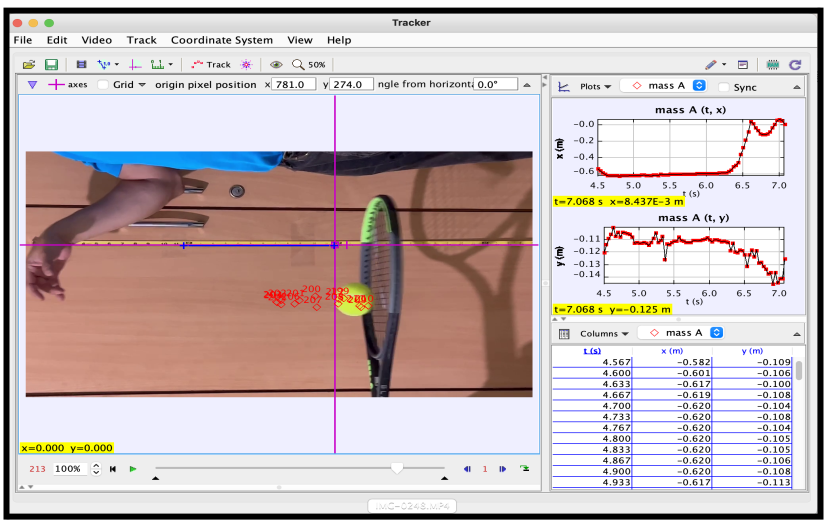

Figure 4: Picture of *Try Tracker* Software Analyzing my Data

10. Processed Data

10.1 Sample Calculation for Angle of 165° and Radius of -3.2 inches

Error Calculation:

$$\text{Average Bounce Height } (h_{average}) = \frac{0.064 + 0.066 + 0.070 + 0.070 + 0.064}{5} \approx 0.067 \; meters$$

$$\text{Absolute Uncertainty } (\Delta h_{average}) = \frac{Max - Min}{2} = \pm 0.003 \; meters$$

$$\text{Coefficient of Restitution (COR)} = \sqrt{\frac{h_{average}}{h_{initial}}} = \sqrt{\frac{0.067}{0.610}} \approx 0.331$$

COR Absolute Uncertainty (ΔCOR) = $\frac{1}{2} * (\frac{\Delta h_{average}}{h_{final}} + \frac{\Delta h_{initial}}{h_{initial}}) \approx \pm 0.023$

Conversion of radius to S.I. units:

Radius (r) = $\frac{-3.2}{39.37} \approx 0.081\ meters$

Radius Absolute Uncertainty (Δr) = $\pm\frac{0.03125}{39.37} \approx \pm 0.001\ meters$

Angle stays the same as it is already in S.I. units

Table 2: Trial Measurements of the Average Bounce Height and COR for respective Angles and Radii of the Impact Points of the tennis balls

Angle Between the Impact Point and the Horizontal Axis of the Tennis Racket (Θ in °) $\Theta \pm$ **0.500°**	**Radius of the Impact Point (r in $meters$)** $r \pm$ **0.001** $meters$	**Average Bounce Height ($h_{average}$ in $meters$)** $h_{average} \pm \Delta h$	**Coefficient of Restitution** $e \pm \Delta e$
1. 72°	1. -0.081	0.130 ± 0.006	0.462 ± 0.022
	2. -0.041	0.152 ± 0.004	0.499 ± 0.014
	3. 0.000	0.126 ± 0.002	0.455 ± 0.007
	4. 0.041	0.075 ± 0.006	0.352 ± 0.037
	5. 0.081	0.066 ± 0.003	0.330 ± 0.023
2. 90°	1. -0.081	0.165 ± 0.004	0.520 ± 0.011
	2. -0.041	0.144 ± 0.003	0.486 ± 0.011
	3. 0.000	0.127 ± 0.000	0.456 ± 0.001
	4. 0.041	0.100 ± 0.004	0.404 ± 0.018
	5. 0.081	0.051 ± 0.003	0.289 ± 0.025
3. 125°	1. -0.081	0.190 ± 0.002	0.558 ± 0.006
	2. -0.041	0.142 ± 0.004	0.482 ± 0.015
	3. 0.000	0.121 ± 0.013	0.445 ± 0.053
	4. 0.041	0.079 ± 0.003	0.359 ± 0.017
	5. 0.081	0.039 ± 0.002	0.252 ± 0.020

165°	1. -0.081	0.067 ± 0.003	0.331 ± 0.023
	2. -0.041	0.116 ± 0.005	0.435 ± 0.020
	3. 0.000	0.126 ± 0.006	0.455 ± 0.023
	4. 0.041	0.090 ± 0.001	0.384 ± 0.006
	5. 0.081	0.026 ± 0.002	0.207 ± 0.029
180°	1. -0.081	0.027 ± 0.004	0.210 ± 0.066
	2. -0.041	0.101 ± 0.001	0.407 ± 0.008
	3. 0.000	0.128 ± 0.006	0.457 ± 0.022
	4. 0.041	0.095 ± 0.002	0.395 ± 0.011
	5. 0.081	0.048 ± 0.002	0.282 ± 0.021

11. Data Analysis

To model my data, I used Python Libraries like *NumPy* and *Matplotlib*. I created a 3D relationship rather than 2D as each impact point equally consists of both an angle and radius. Therefore, measuring the relationship between COR and these individual variables separately would make no sense, as it would not provide a generalized formula for impact points (main objective of this IA. E.g. If I tried to create a function that defined COR vs radius, the COR for the same radius would have multiple different values (making it a meaningless scatterplot).

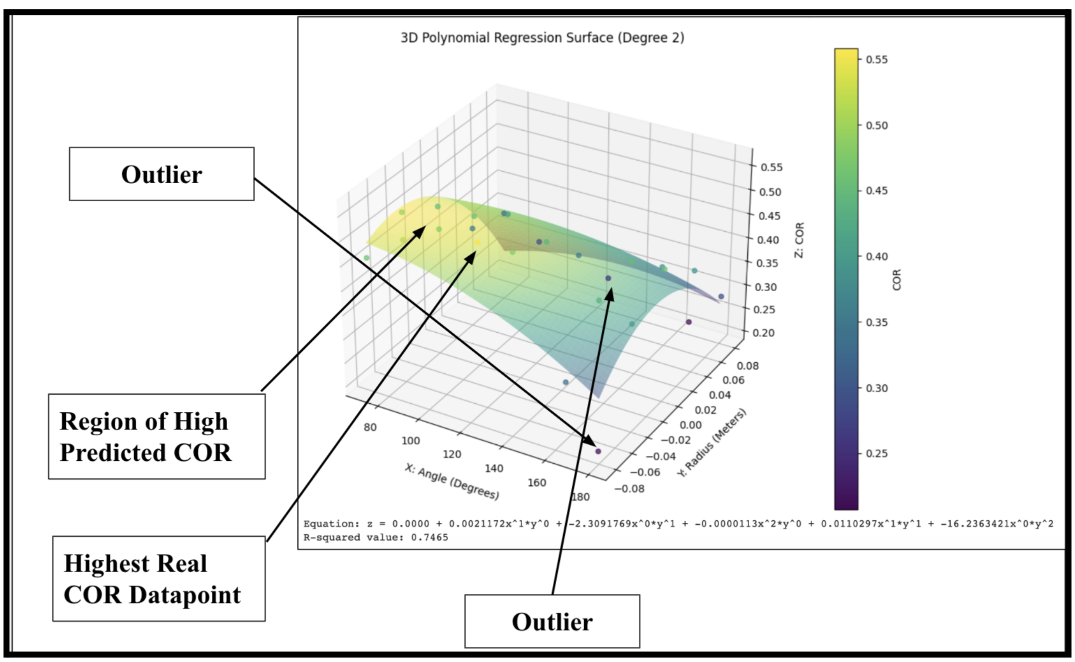

Figure 5: 3-D Graph of Polynomial Relationship between COR, Impact Angle, and Impact Radius

I used 2nd degree polynomial regression to fit my data (in which x and y represented the angle and radius of the impact point respectively, and z represented the COR):

- Equation: $z = 0.0000 + (0.0021172)x - (2.3091769)y + (0.0000113)x^2 + (0.0110297)xy - (16.2363421)y^2$

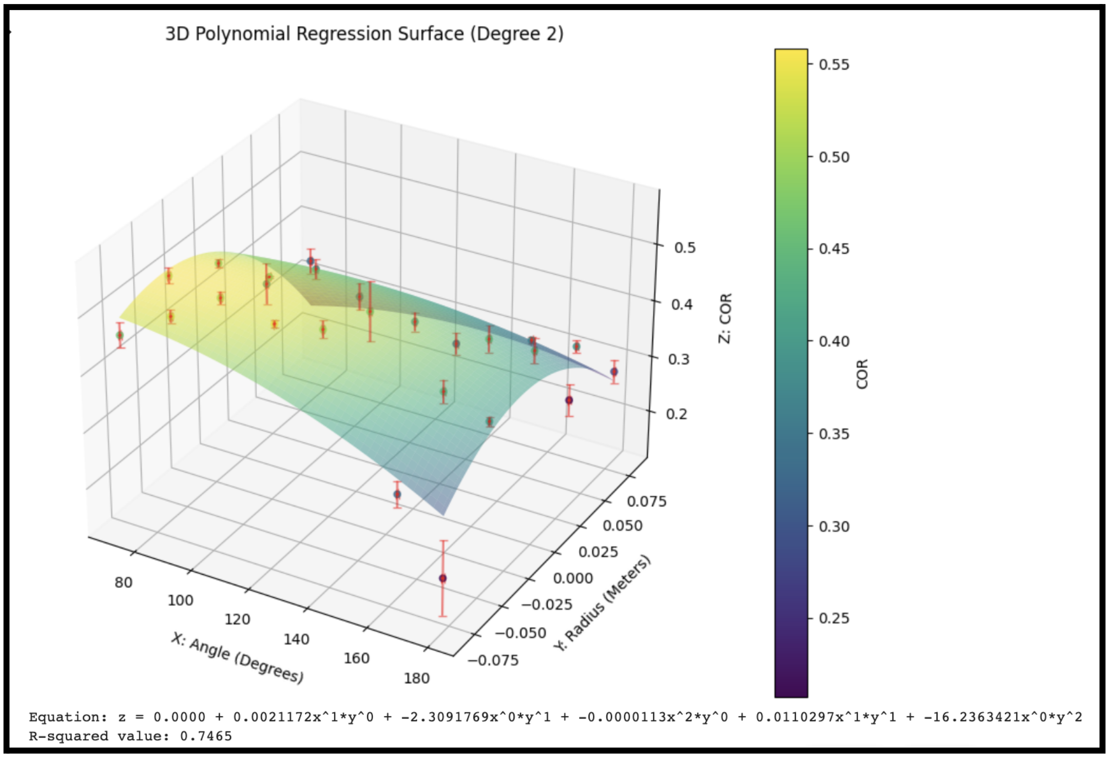

Figure 6: Scatter Plot of Graph with Error Bars

From **Figure 5**, we can notice that the hypothesis generally remains true for my experiment, in the sense that the regions where COR was the highest were regions where either the net tension of strings was higher, or regions which were closer to my racket's vibration node, such as at its center or near its throat. However, my model's values for extreme COR (high and low values) were generally different from the true values of COR, indicating that it perhaps is better at locating regions of high COR rather than predicting the values of actual COR itself. Moreover, the largest COR value from my experimental data (-0.081m, 125°) and one of the smallest COR values from my experimental data (0.081m, 125°) turned out to be outliers in my model, again proving that it struggles to predict extreme data points. These outliers also challenge my specific hypothesis about the maximum region of COR.

While the region of strings directly under the center of a "brand-new" tennis racket would typically be the strongest, I suspect that my many years playing with and fiddling with the strings of the racket used in the experiment must have reduced the net tensile strength in certain areas, possibly even making other areas more powerful and better for hitting shots like volleys or returns of serves. Nevertheless, my model still predicted this region to be one of maximum COR, possibly suggesting that my model would be a better representation for 'brand new' tennis rackets.

I chose to use 2nd-degree polynomial-regression rather than 1st degree (multilinear-regression) and 3rd-degree polynomial-regression because it could produce the most reliable as well as generalized relationship for my experimental data. Multilinear regression simply yielded too little an accuracy rate, whereas 3rd degree polynomial regression simply overfit the data, meaning that it would not produce a relationship of high accuracy across other tennis racket models/ experiments.

Finally, I decided to use R^2 score to determine the accuracy of my relationship as it is unitless and can be easily used to determine the relationship between all my variables. Given that y_i represents the ith experimental value, $\widehat{y_i}$ represents the model's predicted values, and $\bar{y}$ represents the mean of the experimental values, R^2 can be calculated as follows:

$R^2 = 1 - \frac{\Sigma(y_i - \widehat{y_i})^2}{\Sigma(y_i - \bar{y})^2} = 0.7465$, meaning that my model is a relatively accurate and well-generalized representation of my experimental data. [4]

12. Evaluation and Further Investigation

The investigation above answers the research question that the COR of a tennis racket has a **polynomial** relationship with the impact point that a tennis ball hits. However, it has certain flaws.

Weakness	Solution
1. I could not objectively measure the net tension at each impact as I could not afford the instrument required to do so.	Use a "string tension meter" to calculate the net tension at each impact point rather than having to refer to the general regions where tension is higher.
2. I had to use a "used" tennis racket rather than a new one, so I could not completely validate my hypothesis about the maximum point of COR.	Measure the final bounce heights on a brand-new tennis racket so that all the strings are in their normal/ correct configuration.
3. Human error existed when I was dropping the ball (imperfect hand-eye coordination).	A robotic arm that can accurately be centered above an impact point and drop a ball.

[4] "Numeracy, Maths and Statistics." *Academic Skills Kit*, https://www.ncl.ac.uk/webtemplate/ask-assets/external/maths-resources/statistics/regression-and-correlation/coefficient-of-determination-r-squared.html. Accessed 21 Sept. 2023.

To continue to take my Physics IA research forward, I suggest the following:

1. Rather than simply using proportionality to devise a relationship between the impact point and COR of a tennis ball and tennis racket, someone should attempt to discover all the independent variables within an impact point that can affect the COR in order to create a reliable formula for its measurement.
2. A person can also carry out this experiment (and devise a reliable formula to measure COR) across multiple models of tennis balls and tennis rackets to determine how the experiment and results can vary. Perhaps, a database can be created to show all these findings.
3. Finally, a person can carry out this experiment with different racket-based sports, such as squash, etc

13. Conclusion

All in all, my Physics IA successfully answers the research question posed and generalizes a relationship between the COR and different impact points of a tennis ball and tennis racket. With the equation $z = 0.0000 + (0.0021172)x - (2.3091769)y + (0.0000113)x^2 + (0.0110297)xy - (16.2363421)y^2$ and an R^2 of 0.7465, it is evident that the model I have devised can adequately find a mathematical relationship between the majority of data but can sometimes make errors for extrema points. Furthermore, these results are substantiated by already-existing physics principles and tennis knowledge, increasing their validity. However, with additional collection data, perhaps better models can be created to yield more accurate results.

14. References

1. Admin. "Coefficient of Restitution - Definition, Range of Values, Examples, Solved Problems." *BYJU'S*, 13 Apr. 2021, https://byjus.com/jee/coefficient-of-restitution/.
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6. "The Physics of Tennis Racket Sweet Spots." *COMSOL*, 27 Aug. 2015, https://www.comsol.com/blogs/the-physics-of-tennis-racket-sweet-spots/.